Stat 61S: Assignment 2 due Friday, Sept. 11.

1. With acceptance sampling, a purchaser samples 4 items from a lot of 100 and rejects the lot if one or more are defective. Graph the probability that the lot is accepted as a function of the percentage (or count) of defective items in the lot. It's okay to draw the graph by hand with a few points evaluated. Be sure to label your axes.

2. A special deck of cards contains 40 cards, with cards numbered $1, 2, \ldots, 10$ for each of four colors ($10(4) = 40$ cards). Four card are drawn at random without replacement.

   a) Compute the probability you get four cards of the same color.

   b) Compute the probability you get "two pair", meaning two cards with one number and two cards with another different number (e.g., two 1's and two 2's).

   c) Compute the probability you get "three of a kind", meaning three cards with one number and a fourth card with a different number (e.g., three 1's and a 2).

3. A group of 60 second graders is to be randomly assigned to two classes of 30 each. Five of the second graders, Marcella, Sarah, Michelle, Katy and Camerine, are close friends. It may be helpful to label the two classes class 1 and class 2. Then you could define a random variable like $X =$ the number of friends in class 1 (e.g.).

   a) What is the probability the five friends will be in the same class?

   b) What is the probability that exactly four of the friends will be in the same class?

   c) What is the probability that Marcella will be in one class and her friends in the other?

4. A multiple choice test consists of $n = 25$ items, each with four choices. A passing grade is 15 out of 25.

   a) Suppose a student can correctly eliminate two of the choices on each item and guesses randomly at the other two choices. Use Table 1 to find the probability the student gets exactly 15 items correct, and the probability the student passes.

   b) Answer the questions in part a again, this time assuming the student can eliminate only one choice on each item and guesses at the other three choices. The table will not work for this situation so you will need to use the formula - show your work. This will be tedious, but it is fine if you get the answer to within 0.0001 of the exact value (so you do not need to evaluate all of the terms in the sum). Part of my reasoning for having you go through this is to help you appreciate the ease and accuracy of the approximations we will learn.

5. The university administration assures a professor that there is only a 1 in 10,000 chance of her being trapped in a much-maligned elevator in the mathematics building. Assume this is the probability on each elevator ride, and that the outcomes on different days are all independent (a dubious assumption in practice). Suppose she goes to work 5 days a week, 52 weeks a year for 10 years, and always rides the elevator up to her office when she arrives (and takes the stairs down).

   a) Find the exact probability that she will never be trapped, that she will be trapped exactly once, and that she will be trapped exactly twice.

   b) Repeat these computations using the Poisson approximation to the Binomial (for large n and small p).

6. Consider a sequence of independent Bernoulli trials, each with probability of success θ.

   a) Let X represent the number of successes before the first failure (this is a variation of the Geometric distribution defined in 2.1.3). Write out the frequency function for X (be sure to specify the possible values X can take on).

   b) Let Y represent the number of successes before the rth failure (this is a variation of the Negative Binomial distribution in 2.1.3). Write out the frequency function for Y.

7. Using the Geometric distribution defined in part a of the previous problem, show that

$$P(X \geq n + k \,|\, X \geq n) \;=\; P(X \geq k).$$

   Explain why this makes sense intuitively, given the way X is defined.

8. Phone calls are received at a certain residence according to a Poisson process with rate $\lambda = 2$ calls per hour.

   a) If Diane takes a 10 minute shower, what is the probability the phone rings during that time?

   b) How long can her shower be if she wishes the probability of receiving no phone calls to be at least 0.5?

9. Suppose that X is a continuous random variable with density function $f_x(x) = cx^3$, for $0 \leq x \leq 1$ and $f_x(x) = 0$ otherwise.

   a) Find the value of the constant c.

   b) Find the cumulative distribution function (CDF) for X.

   c) Find $P(0.1 \leq X \leq 0.5)$.

10. Suppose the lifetime in years of an electric component follows an exponential distribution with $\lambda = 0.1$.

   a) Find the probability that the lifetime is less than 10 years.
   b) Find the probability the lifetime is between 5 and 15 years.
   c) Find the value t such that the probability that the lifetime exceeds t is 0.01.